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# Static Analysis of Pure Epoxy Beam Induced by Piezoelectric Actuator by using Detailed Models

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**Abstract—***In this paper***,** *three analytical models and one approximating model depicting the detailed mechanics of induced strain actuators which is surface bonded to the one-dimensional structures are obtained. The models demonstrate the extension ,bending, and local shear deformations induced. The objective is to find the slope and deformation for three analytical model and one approximating models and compared for the induced strain actuation of the pure epoxy beam.*

**Nomenclature**

| *symbol* | *Definition* |
|---|---|
| $F_{bl}$ | Blocked force |
| $d_{31}$ | Piezoelectric constant |
| $E_c$ | Modulus elasticity of actuator |
| $B_c$ | **W**idth of actuator |
| $V$ | Electric field |
| $B_b$ | Width of beam |
| $F$ | Actuation force |
| $EA_b$ | Extensional stiffness of the beam |
| $EA_c$ | Extensional stiffness of two actuator |
| $\epsilon_b$ | Surface strain of the beam |
| $M_{bl}$ | Blocked moment |
| $M$ | Actuation moment |
| $EI_b$ | Bending stiffness of the beam |
| $EI_c$ | Bending stiffness of the two actuator |
| $\epsilon_b^s$ | Surface strain at the top of the beam |
| $\epsilon_b^{-s}$ | Surface strain at the bottom of the beam |
| $\wedge$ | Free strain |
| $\psi_e$ | Stiffness parameter for the extension |
| $\psi_b$ | Stiffness parameter for the bending |
| $\Gamma$ | Shear lag parameter |
| $k$ | curvature |
| $\in$ | Axial strain |
| $(ES)_{total}$ | Coupling stiffness |
| $\theta_b$ | Thickness ratio |
| $K_{ij}$ | Stiffness marix |
| $Q_{\wedge_i}$ | Forcing vector |

## 1. INTRODUCTION

To controlling the structural deformations, there is one approach is to access the one-dimensional structure elements in which actuation strain can be controlled. Actuation strain is defined as the strain is produced by applying electric potential. In this paper, actuation strain is produced by piezoelectric materials. The Free strain $\wedge$ is obtained by modelling the structures which is bonded with the piezoelectric materials. Crawley et al. 1988 studied for induced strain actuators which is used as highly distributed actuators in intelligent structures. With such distributed actuators, it is possible to design structures with inherent vibration and shape control capabilities.

The aim of this paper is to acquire accurate detailed models of the collaboration between the induced strain actuators and one-dimensional structures to which they are bonded. The analytical models are derived in terms of free strain to make these model applicable.

Piezoelectricity is the mechanis that isused for induced strain actuation in which strain creates by applying the electric field. In this paper, PZT-5H has beenused as piezoelectric actuators in applications with the beam (Fansom and Caughey, 1987). It is essential to describe the piezoelectric actuation strain to predict structural deformation by inducing strain with the help of the piezoelectric actuator.

## 2. INDUCED STRAIN ACTUATION OF BEAM

The objective of this paper is to acquire accurate detailed models of the collaboration between the induced strain actuators and one-dimensional structure to which they are bonded. When piezoelectric actuators are coupled to the structure, the actuators and structure mar extend, bend and shear. The comparative importance of these three modes of deformations depend on the geometry and the relative stiffnesses of the actuators, structure and bonding layers. Four models are derived to examine the detailed deformations –

three analytical models and one approximating model. These models are compared to determine what situations higher complexity is needed.

The first model-strain model applicable only to surface bonded actuators and it assumes only uniform extensional strain in the actuator. The second model- Euler Bernoulli model is applicable for either surface bonded or embedded,which includes extension and bending in the actuator.

Initially it was unclear that for a surface bonded actuator to the beam,linear strain distribution of the Bernoulli-Euler model was suitable. Moderately to resolve this uncertainty the approximation model-Galerkin method includes extension and bending in the actuator and host structure was developed.

**a). Simple Blocked Force model**

This blocked force model is an approach to estimate beam response due to induced strain actuation. It is a highly approximate model. The actuator is idealized as a line force and this model does not include any spanwise variation of stress or strain at the actuator location. To achieve the pure extension in the beam the same voltage is applied to the top and bottom actuators.

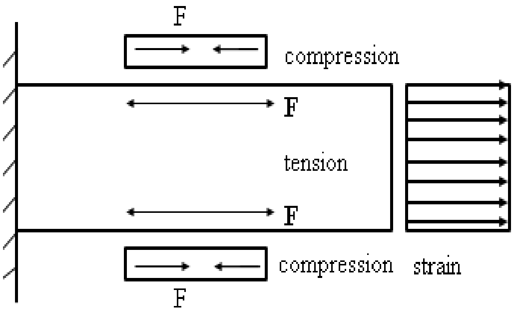


**Figure 1: Simple blocked force model (Pure extension)**

The actuation force in the beam is 2F. The maximum force or blocked force in the direction '1' is:

$$F_{max} = d_{31}E_cB_cV = F_{bl} \tag{1}$$

$$F = \frac{d_{31}\frac{V}{t_c}}{\frac{2}{E_bT_bB_b}+\frac{1}{E_cB_cT_c}} \tag{2}$$

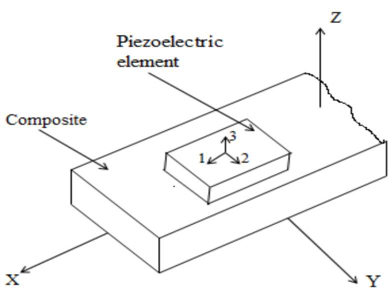


**Figure 2: Represent of the direction of Beam and PZT**

$$F = F_{bl}\frac{EA_b}{EA_b+EA_c} \tag{3}$$

The strain distribution across the beam thickness is uniform.

$$\epsilon_b = \frac{2F}{EA_b} \tag{4}$$

For the case of pure bending, an equal but opposite voltage is applied to the top and bottom actuators. The strain is varying linearly across the thickness of the beam. It is assumed as there is no variation of bending stress along the length of the actuator.

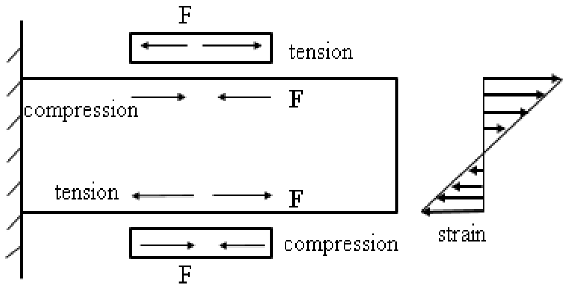


**Figure 3: Simple blocked force model (Pure Bending)**

The actuation moment can be calculated as:

$$M = M_{bl}\frac{EI_b}{EI_b+EI_c} = M_{bl}\frac{EA_b}{EA_b+3EA_c} \tag{5}$$

And for this case, the beam axial strain varies linearly across the beam thickness as shown in Figure. The surface strain for the top of the beam is:

$$\epsilon_b^S = -\wedge\frac{3EA_c}{EA_b+3EA_c} \tag{6}$$

And for the bottom of the beam, the surface strain is:

$$\epsilon_b^{-s} = -\wedge\frac{3EA_c}{EA_b+3EA_c} \tag{7}$$

**b). Uniform Strain model**

In this model, the bond layer between the piezo actuator and the structure has a finite stiffness. Strain generated by the piezo is dissipated in the deformation of the bond layer itself. For the perfectly-bonded actuator, the shear is concentrated near the edge of the actuator. For the case of pure extension induced by a pair of actuators , the induced strain is:

$$\epsilon_c = \epsilon_b = \frac{2\wedge}{2+\psi_e} \tag{8}$$

Where $\psi_e = \frac{(EA)_b}{(EA)_c}$

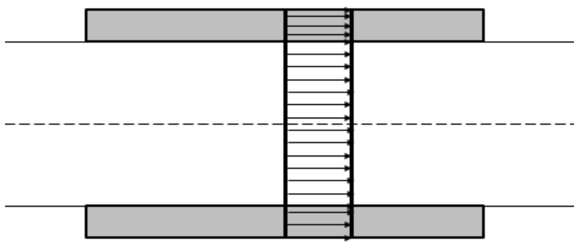

**Figure 4: Uniform strain Extension**

For the case of pure induced bending by a pair of actuators, the strain at the upper surface of the beam is:

$$\epsilon_c = \epsilon_b^{surf} = \frac{6\wedge}{6+\psi_b} \quad (9)$$

Where $\psi_b = \frac{12(EI)_b}{t_s^2(EA)_c}$

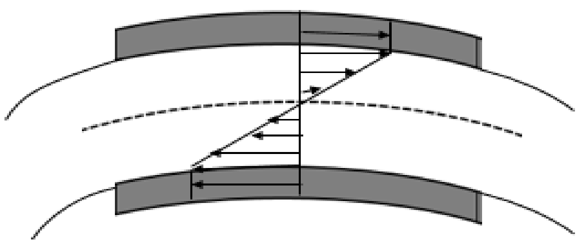

**Figure 5: Uniform strain Bending**

The shear lag parameter is a measure of the effective stiffness of the bond.

$$\Gamma^2 = \frac{L_c^2}{4}\frac{B_c G_s}{EA_b t_s}\left(\alpha + \frac{E_b A_b}{E_c A_c}\right) \quad (10)$$

**c). Euler-Bernoulli model**

This model gives more accurate results than uniform strain model, especially for thin bond layers. There is a linear distribution of strain in the cross-section for both the actuator and host structure and there is no variation of transverse displacement ($w$) across the thickness. E-B model considers the actuators as an integral part of the structure. The axial strain varies linearly through the thickness according to

$$\in = \in_0 - zk \quad (11)$$

From the piezoelectric constitutive relations, the stress in

the layer of the beam is given by:

$$\sigma(z) = E(Z)[\in(Z) - \wedge(Z)] \quad (12)$$

$$(EA)_{total} = \int E(z)B(z)dz \quad (13)$$

$$(ES)_{total} = -\int E(z)B(z)dz \quad (14)$$

$$(EI)_{total} = \int E(z)B(z)Z^2 dz \quad (15)$$

$$(EA)_{total} \in_0 + (ES)_{total} w'' = F + F_\wedge \quad (16)$$

$$(ES)_{total} \in_0 + (EI)_{total} w'' = M + M_\wedge \quad (17)$$

For pure extension case, the axial strain of the beam is:

$$\in_0 = \frac{EA_c}{EA_b + EA_c}\wedge \quad (18)$$

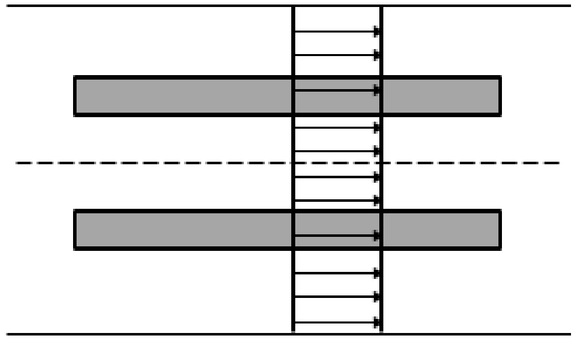

**Figure 6: Bernoulli-Euler Extension**

For pure bending case, the axial strain distribution is:

$$\in(z) = -z\frac{M_\wedge}{(EI)_{tot}} = -\frac{6\left(1+\frac{1}{\theta_b}\right)\frac{2}{t_b}\wedge}{(\psi+6)+\frac{12}{\theta_b}+\frac{8}{\theta_b^2}}z \quad (19)$$

Where $\theta_b = \frac{t_b}{t_c}$

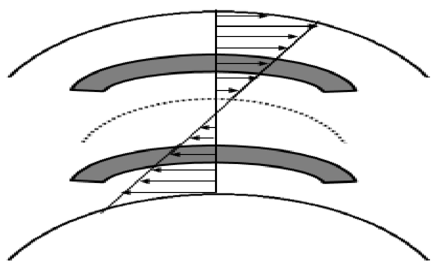

**Figure 7: Bernoulli-Euler Bending**

**d). Approximate Solution-Galerkin Method**

For the Galerkin solution, the response is assumed to be a summation of functions and it must separately satisfy all boundary conditions i.e; geometric and forced –boundary conditions.

Strains and curvatures expressing in terms of displacements,

$$\begin{Bmatrix}\varepsilon_o \\ k\end{Bmatrix} = \begin{bmatrix}\frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial^2}{\partial x^2}\end{bmatrix}\begin{Bmatrix}u_0 \\ w\end{Bmatrix} \quad (20)$$

Assuming the displacement distribution in terms of functions such as:

$$u_0(x) = \sum_{I=1}^{M} \emptyset_{ui}(x) q_i \quad (21)$$

$$w(x) = \sum_{J=1}^{N} \emptyset_{wj}(x) q_{j+M} \quad (22)$$

For pure induced extension, consider the case of symmetric configuration $(ES)_{tot} = 0$.

$$K_{ij} = \int_0^{l_b} \emptyset_{u_i} \frac{\partial}{\partial x}\left(EA_{tot}\frac{\partial \emptyset_{u_i}}{\partial x}\right) dx \quad (23)$$

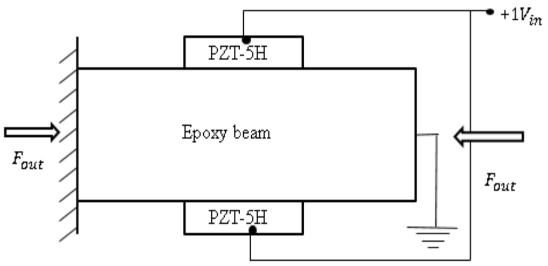


**Figure 8: Configuration for Pure Extension**

$$Q_{\wedge_i} = \int_0^{L_b} \emptyset_{u_i} \frac{\partial F_\wedge}{\partial x} dx \quad (24)$$

For pure induced bending, consider the case of symmetric configuration $(ES)_{tot} = 0$.

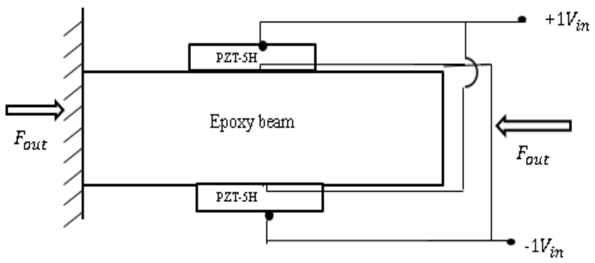


**Figure 9: configuration for Pure Bending**

$$K_{ij} = \int_0^{l_b} \emptyset_{w_i} \frac{\partial^2}{\partial x^2}\left(EI_{tot}\frac{\partial^2 \emptyset_{w_i}}{\partial x^2}\right) dx \quad (25)$$

$$Q_{\wedge_i} = \int_0^{L_b} \emptyset_{w_i} \frac{\partial^2 M_\wedge}{\partial x^2} dx \quad (26)$$

## 3. NUMERICAL SIMULATION FOR STATIC RESPONSE

The flexural deflections are calculated for cantilever epoxy beam which is surface bonded furnished with the PZT-5H actuators. The dimension of the epoxy beam is length 609.6mm, width 50.8mm and thickness 0.8mm[4]. The properties of actuator PZT-5H are piezoelectric constant $d_{31}$ = –274e-12 V/m, length 50.8mm, width 25.4mmand thickness of piezoelectric actuator is 0.32mm[4] and the applied voltage is 1V.

The flexural deflections of beam agitated by various dimensions of PZT-5H actuators were computed using the created MATLAB code.

**Table I Properties of Epoxy Beam and Actuator**

| Properties | Carbon/Epoxy | Actuator |
|---|---|---|
| E | 3.5 GPa | 72.4GPa |
| nu | 0.28 | 0.3 |

**Table II Induced strain for Pure Extension**

| Model | Strain value |
|---|---|
| Simple blocked force method | -0.7639 $\mu\epsilon$ |
| Uniform Strain model | -0.79835$\mu\epsilon$ |
| Euler-Bernoulli model | -0.7639 $\mu\epsilon$ |

**Table III Induced strain for Pure Bending**

| Model | Strain value |
|---|---|
| Simple blocked force method | -0.82304$\mu\epsilon$ |
| Uniform Strain model | -0.82304$\mu\epsilon$ |
| Euler-Bernoulli model | -0.58372$\mu\epsilon$ |

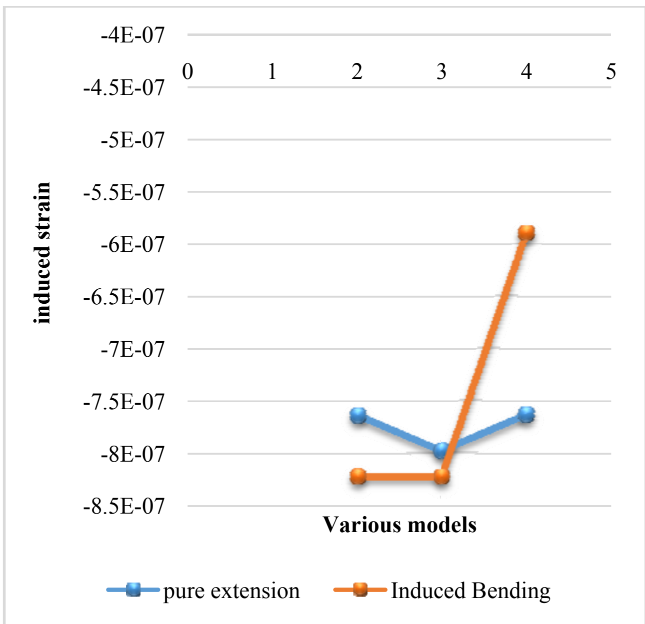


**Figure 10: Induced strain for Pure Extension and Pure Bending**

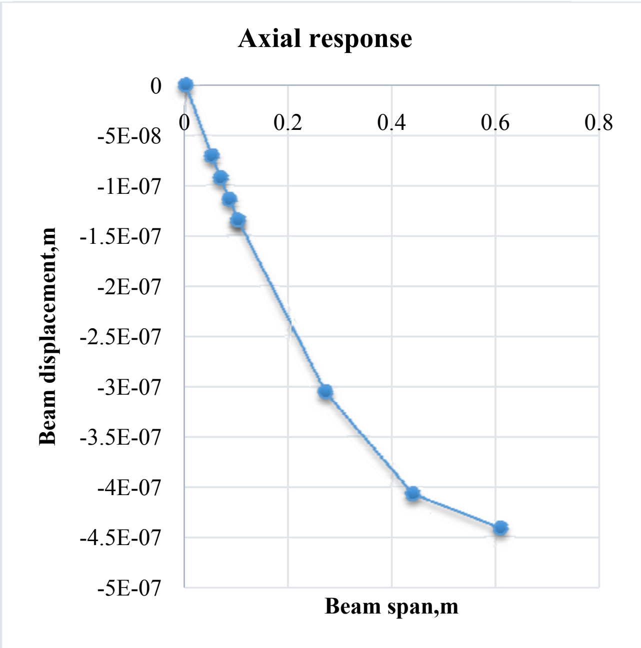


**Figure 11: Axial response by using the Galerkin method**

**Table IV Tip slope and tip deflection for the analytical model**

| **Model** → | **Blocked force** | **Uniform strain** | **Euler model** |
|---|---|---|---|
| Tip slope ($10^{-4}$rad) | -1.0453 | -0.4867 | -0.6054 |
| Tip displacement | -0.557e-4 | -0.2769e-4 | -0.6961e-3 |

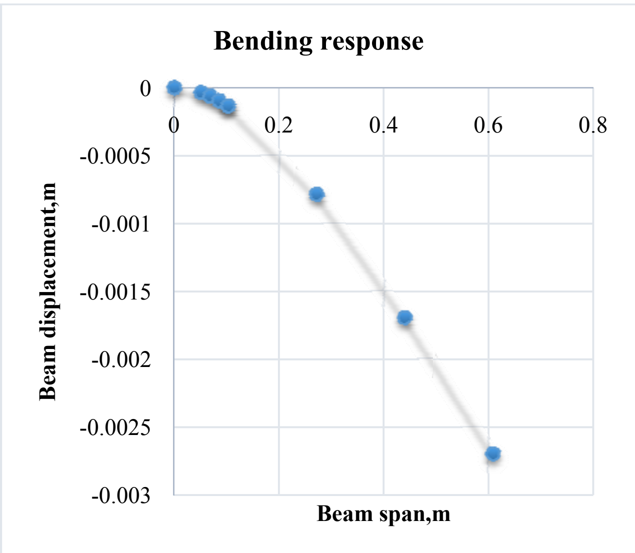


**Figure 12: Bending response by using the Galerkin method**

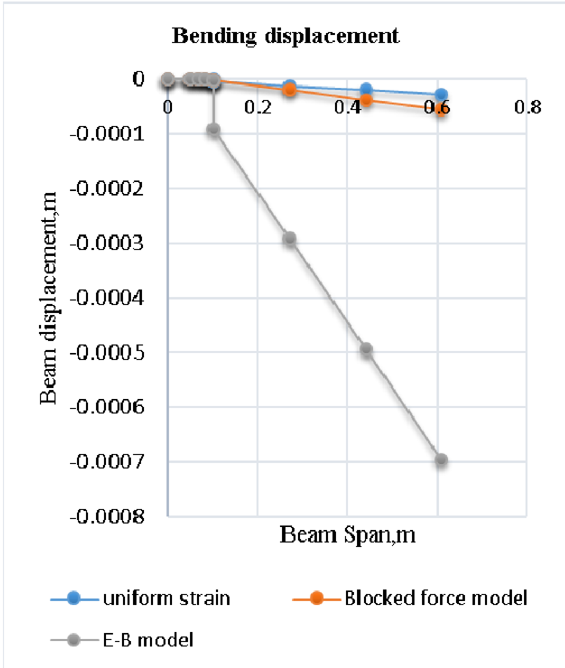


**Figure 13: Bending displacement for various model**

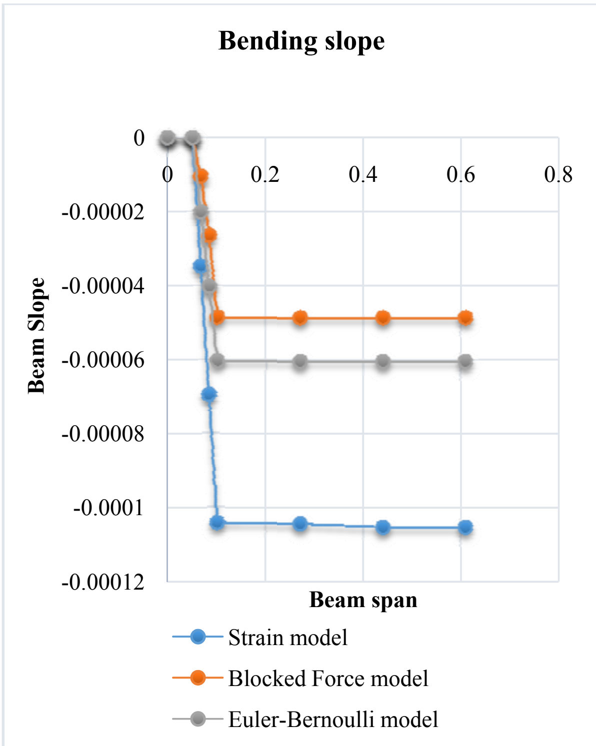


**Figure 14: Bending Slope for various model**

## 4. CONCLUSIONS

In this study, four separate models of induced actuation strain for one-dimensional structure have been developed. Piezoelectric actuators were symmetrically surface bonded on an epoxy beam to obtain the flexural deflection and slope by applying the electric field. From the results, it was concluded the following observations which are following as:

- For pure extension, induced strain predicted by the simply blocked force and Bernoulli model are identical.
- For pure bending, induced strain predicted by the simply blocked force and uniform strain model are identical.
- Uniform strain theory predicted lower deflections among the other theory.
- Simple blocked force theory predicted lower slope among the other theory.
- Based on comparison with the Galerkin and the analytical models, Euler- Bernoulli model was estimated to accurately predict the deformation.

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